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WAVE RADIATION FROM SLENDER BODIES

by

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Wave radiation by a slender body is discussed, with emphasis on the scale of the characteristic wavelength in relation to the disparate width and length of the body. The discussion is illustrated by reviewing the analysis of oscillatory forced motions of a slender floating body in otherwise calm water. Two complementary regimes are considered, where the wavelength is long (comparable to the body length) or short (comparable to the beam). The limitations of these two theories suggest the need for a unified approach, where geometric slenderness is assumed without restricting the wavelength. A possible approach to the unified theory is outlined in general terms, and illustrated in detail by considering the simpler physical problem of axisymmetric acoustic radiation by a slender body in an unbounded medium.

Reinier Timman influenced the field of applied mathematics by his own work, that of his students at Delft, and through the work of many others who were his students in a generalized sense. The occasional opportunities to discuss my own work with Professor Timman were invaluable. Traces of his stimulating insight are present in each contribution to this Memorial Symposium, which serves both to honour and to perpetuate his unique influence.

1. INTRODUCTION

The field of slender-ship theory is a logical development from the earlier analysis of slender bodies in aerodynamics. However a significant complication results for ships and other floating slender bodies, due to the extra length scale associated with the characteristic wavelength (λ), which must be related to the disparate length (L) and beam (B) of the vessel. An analogous situation exists for

acoustic radiation in an unbounded medium. With the fundamental geometric slenderness parameter $B/L=\varepsilon<<1$, we seek an asymptotic solution valid to leading order in ε. For this purpose it generally is necessary to restrict the order of magnitude of the wavelength to one of two complementary regimes. For long waves, comparable to the body length, $\lambda/L=O(1)$, whereas in the short-wavelength case $\lambda/B=O(1)$. (Some workers reserve the term "slender-body theory" or "slender-ship theory" for the long-wavelength regime. A more liberal definition is adopted here, the word "slender" being applied only to the geometry of the body.)

In the long-wavelength regime, interactions are significant between adjacent sections of the body, but wave effects are absent from the near field close to the body surface. For the ship-motion problem, the importance of transverse gradients in the inner region is such that the linear free-surface boundary condition degenerates to the "rigid-wall" condition of zero vertical velocity. This greatly simplifies the theory, but also restricts its domain of applicability to sub-resonant frequencies.

In the short-wavelength regime wave effects are present in the near field, but one recovers a simple strip theory without interactions between sections. In one sense this result is very satisfying, as it provides some rigor to the otherwise empirical strip theory of ship motions. However, the consistent leading-order theory is again oversimplified for practical purposes. In particular, the hydrodynamic effects of the ship's forward speed are absent completely from the leading-order theory, as suggested by Vossers (1962) and confirmed by Joosen [1964] and Ogilvie and Tuck (1969).

Faced with the choice between these two complementary theories, and the observation that practical ship motions occur in wavelengths which generally are intermediate between the length and beam, it is natural to seek a "unified" slender-body theory where the wavelength is unspecified in advance. A suggested approach to that objective is the principal contribution of this lecture.

First, I shall outline briefly the separate treatments of long- and short-wavelengths for the ship-motion problem. A possible approach is then developed for the more general case, and illustrated by treating the analogous problem of acoustic radiation from a slender body in an unbounded medium.

For pedagogic reasons the discussion will be simplified as much as possible. A significant restriction is to consider only the radiation problem, with a slowly-varying distribution of the prescribed normal velocity on the body surface. The diffraction problem for scattering of incident waves can be solved in a similar manner if the wavelength is large, but for short wavelengths the diffraction problem is more difficult.

Another restriction is to neglect forward speed in the ship-motion problem. This simplifies the boundary conditions of the problem, and the resulting solution. The effects of forward speed are included in the long- and short-wavelength problems, respectively, by Newman and Tuck (1964) and Ogilvie and Tuck (1969). No fundamental difficulties are anticipated in adopting the suggested unified approach to include forward-speed effects.

2. THE SHIP-MOTION PROBLEM

The problem of interest here is to analyse the motions of a ship, or more generally of any elongated vessel floating on the free surface. Cartesian coordinates (x,y,z) are defined with $z=0$ the plane of the undisturbed free surface, and the direction of the positive z-axis upward. It is convenient to nondimensionalize and orient these coordinates such that the longitudinal body axis occupies the segment $(0,1)$ of the x-axis.

The motions of the body and surrounding

fluid are assumed to be oscillatory in time, with the complex factor $e^{i\omega t}$ suppressed. The motions are assumed also to be of small amplitude by comparison to the wavelength and body dimensions. Thus a linearized boundary-value problem can be justified, after making the usual assumption that the fluid is ideal and incompressible.

The fluid velocity vector is expressed as the positive gradient of a potential $\phi(\underline{x})$, which is governed throughout the fluid domain by the Laplace equation

$$\nabla^2 \phi = \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \quad . \tag{1}$$

The two principal boundary conditions are on the wetted surface of the body and on the free surface. The appropriate boundary condition on the body surface S is

$$\partial\phi/\partial n = V_n \quad , \tag{2}$$

where V_n is the specified normal velocity. In the linearized theory, (2) is imposed on the mean position of S, and the free-surface boundary condition takes the form

$$K\phi-\phi_z = 0, \quad \text{on } z = 0 \ , \tag{3}$$

with $K=\omega^2 L/g$. For deep water the fluid motion should vanish as $z \to -\infty$, and $K=2\pi L/\lambda$ is the (nondimensional) wavenumber of the radiated waves.

This boundary-value problem is completed by imposing the radiation condition

$$\phi \sim R^{-1/2}\, e^{-iKR} \ , \quad \text{as} \quad R=(x^2+y^2)^{1/2} \to \infty \quad . \tag{4}$$

Alternatively, the radiation condition (4) can be deleted if the steady-state oscillatory time-dependence is replaced by a suitable initial-value problem, with a state of rest at $t=-\infty$. A convenient approach is to retain the time-dependence $e^{i\omega t}$, but with a complex frequency ω having a vanishingly small *negative* part,

$$\omega \to \omega - i0 \quad . \tag{5}$$

From the definition of the wavenumber it follows that

$$K \to K - i0 \quad . \tag{6}$$

We shall use (6) in place of (4), since this avoids the asymptotic analysis to verify the far-field wave form. Furthermore, (5) and (6) can be utilized in the acoustic problem of Section 3.

The body geometry is characterised by a length $L \equiv 1$, beam (B) and draft (T), such that $(B,T) = O(\varepsilon)$. Our approach is based on the method of matched asymptotic expansions. Thus, we define an inner region adjacent to the body surface, where $(y,z) = O(\varepsilon)$, and an outer region where $(y,z) = O(1)$. By assumption there exists an overlap region $\varepsilon \ll (y,z) \ll 1$, where the solutions obtained separately in the inner and outer regions are both valid and can be matched. This technique is described by Van Dyke (1975), and was first applied to slender ships by Tuck (1963).

In the inner region, a coordinate-stretching argument can be used to show that gradients in the transverse plane $(\partial/\partial y, \partial/\partial z)$ are $O(\varepsilon^{-1})$, and dominate the longitudinal derivative $\partial/\partial x = O(1)$. Thus (1) reduces to the two-dimensional Laplace equation

$$\phi_{yy} + \phi_{zz} = 0 \ , \tag{7}$$

with the error a factor $1+O(\varepsilon^2)$. The solution in the inner region is of the form

$$\phi(x,y,z) = \phi^{(2D)}(y,z;x) + f(x) \quad . \tag{8}$$

Here $\phi^{(2D)}$ denotes the solution of a two-dimensional boundary-value problem, and $f(x)$ is a trivial solution of (7) which is determined ultimately by matching to the outer solution.

In the outer region there is no preferred direction, and all three components of the gradient operator are of the same order of magnitude. Thus, the three-dimensional Laplace equation (1) holds in the outer region, without simplification. From geometric considerations, the components of the unit normal vector $\underline{n}$ directed into the body surface are

$$n_x = O(\varepsilon) \ , \qquad (9)$$

$$(n_y, n_z) = O(1) \ . \qquad (10)$$

The estimates (9-10) must be modified near the bow and stern unless these ends are sharply pointed. Hereafter we restrict the body geometry such that (9-10) are uniformly valid along the length of the ship. Furthermore, a longitudinal source distribution will be utilized subsequently to derive the outer solution, and it will be necessary to restrict the source strength near the ends. We shall assume that the source strength is zero at the body ends, especially in connection with various partial integrations; a stronger condition may be necessary, particularly if the solution is to be uniformly valid near the ends. This question is discussed by Ursell (1962), but is not of practical importance in the leading-order analysis of vertical ship motions.

It is obvious that the body boundary condition (2) is applied in the inner region, and the radiation condition (4) is valid only in the outer region. The free-surface condition (3) must be applied in both the inner and outer regions, and the manner in which this is done depends on the order of magnitude of the wavelength λ or wavenumber K.

<u>Long Wavelengths, K=O(1)</u>

If $K=O(1)$, the dominance of transverse gradients in the inner region leads to the "rigid" free-surface condition

$$\phi_z = 0 \quad \text{on} \quad z=0 \ , \qquad (11)$$

in place of (3). By reflection about the plane $z=0$, $\phi^{(2D)}$ is the solution of a two-dimensional flow exterior to the wetted contour C of the ship and its image $\bar{C}$ above $z=0$, with prescribed normal velocity V_n on C and the same normal velocity on $\bar{C}$.

From conservation of mass $\phi^{(2D)}$ will include a source strength proportional to the net flux associated with the normal velocity V_n on C. Thus for heave and pitch

$$\phi^{(2D)} \simeq \frac{1}{2\pi} \sigma(x) \log r \ ,$$

$$r \equiv (y^2+z^2)^{1/2} >> \varepsilon \ . \qquad (12)$$

The source strength $\sigma(x)$ is the product of the local half-beam and vertical velocity.

The remaining motions of the ship will not result in a net source strength and

$$\phi^{(2D)} \simeq \frac{1}{2\pi} \underline{\mu}(x) \cdot \underline{\nabla} \log r, \quad r >> \varepsilon \ . \qquad (13)$$

For sway and yaw the dipole moment $\underline{\mu}(x)$ can be related to the added mass of $C+\bar{C}$.

An arbitrary "constant" $f(x)$ can be added to $\phi^{(2D)}$, and this constant contributes to the pressure in the inner region adjacent to the body surface. This affects the vertical force on the profile C, but not the horizontal force. This constant is significant for pitch and heave, and must be determined by matching (12) with the outer solution. By comparison, for sway and yaw, the constant can be neglected and the local solution in the inner region is given simply by the two-dimensional potential of the form (13).

Hereafter we consider only pitch and heave. In the outer region a distribution of three-dimensional wave sources is required, governed by the complete free-surface condition (3). The necessary source potential or Green function is well known, cf. Wehausen and Laitone (1960, eq. 13. 17). For a source of complex time-dependence $e^{i\omega t}$, situated at the point $(\xi,0,0)$, the velocity potential can be expressed in the form $G(x-\xi,y,z)$, where

$$G(x,y,z) = \int_0^\infty \frac{k'dk'}{k'-K} \cdot J_0\left[k'(x^2+y^2)^{1/2}\right]e^{k'z} \quad . \qquad (14)$$

Note that (14) satisfies the (three-dimensional) Laplace equation (1), and free-surface condition (3). In view of (6) the appropriate contour of integration is *above* the pole at $k'=K$. With this choice of contour (14) can be shown by asymptotic expansion to *satisfy* the radiation condition (4). Near the source point, (14) is dominated by the singularity

$$G \cong \int_0^\infty dk' \; J_0\left[k'(x^2+y^2)^{1/2}\right]e^{k'z} = (x^2+r^2)^{-1/2} \quad . \qquad (15)$$

In the outer region far from the surface of the body, the potential can be expressed as a source distribution on the body axis with unknown strength $\sigma(x)$, in the form

$$\phi = -\frac{1}{4\pi}\int_0^1 \sigma(\xi)\; G(x-\xi,y,z)\; d\xi \; . \qquad (16)$$

In our subsequent analysis it will be assumed that σ is a regular function, which vanishes at the body ends.

The inner expansion of (16), for small (y,z), is required for matching with the inner solution. This nearfield approximation of (16) can be derived in a systematic manner using Fourier transforms, following Ursell (1962). With an asterisk (*) used to denote the Fourier transform with respect to x, we define

$$\phi^*(y,z;k) \equiv \int_{-\infty}^{\infty} e^{ikx}\, \phi(x,y,z)\; dx \; , \qquad (17)$$

and similarly for other functions of x. By the convolution theorem

$$\phi^* = \frac{1}{4\pi}\,\sigma^*\, G^* \quad , \qquad (18)$$

where G^* is the Fourier transform of (14).

After substitution of the integral representation for J_0, the Fourier transform of (14) is given in the form

$$G^* = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\int_0^\infty \frac{k'}{k'-K}\int_0^{2\pi} e^{k'z + ik'(x\cos\theta + y\sin\theta)}\, d\theta\; dk'dx \quad . \qquad (19)$$

The x-integral in (19) yields a delta function. After an application of generalized harmonic analysis it follows that

$$G^* = \int_{-\infty}^{\infty} \frac{\exp[z(k^2+\kappa^2)^{1/2} + i\kappa y]}{(k^2+\kappa^2)^{1/2} - K}\; d\kappa \qquad (20)$$

where

$$\kappa = k' \sin\theta \quad .$$

An expansion of (20) in modified Bessel functions of argument $(|k|r)$ is derived by Ursell (1962). For small values of this argument the leading-order terms are

$$G^*(r\sin\theta, -r\cos\theta; k) = -2\log\left\{\tfrac{1}{2}|k|r\right\} - 2\gamma + \frac{2K}{(k^2-K^2)^{1/2}}\begin{Bmatrix}\pi - \cos^{-1}(K/|k|) \\ -i\pi - \cosh^{-1}(K/|k|)\end{Bmatrix} \; , \quad \text{for } \begin{Bmatrix}|k| \gtrless K\end{Bmatrix} \; . \qquad (21)$$

(Ursell's expression is conjugate to (21), due to the opposite convention for the complex time dependence, and includes higher-order terms proportional to Kr.)

The inverse transform of (21) can be evaluated, following Ursell (1962). After a second application of the convolution theorem, the inner expansion of the outer solution can be expressed in the alternative forms

$$\phi \cong \frac{1}{2\pi}\,\sigma(x)\log r + \frac{1}{4\pi}\int_0^1 \sigma'(\xi)L_o(x-\xi)\,d\xi + \frac{K}{4\pi}\int_0^1 \sigma(\xi)\,L_1(Kx-K\xi)\,d\xi\,, \qquad (22a)$$

or

$$\phi \cong \frac{1}{2\pi}\,\sigma(x)\left[\log(Kr) + \gamma + \pi i\right] + \frac{1}{4\pi}\int_0^1 \sigma'(\xi)\,L_2(Kx-K\xi)\,d\xi \qquad (22b)$$

Here the kernels L_n are defined as

$$L_o(u) = -\log(2|u|)\,\mathrm{sgn}(u), \qquad (23a)$$

$$L_1(u) = \frac{\pi}{2}\left[H_o(|u|) + Y_o(|u|) + 2i\,J_o(|u|)\right], \qquad (23b)$$

$$L_2(u) = \left[-\log 2|u| - \gamma - \pi i + \int_0^{|u|} L_1(u')\,du'\right]\mathrm{sgn}(u)\,, \qquad (23c)$$

and, in the usual notation, H_o, Y_o and J_o denote the Struve and Bessel functions of order zero. The last term in (22a) vanishes as $K \to 0$, leaving the result for a slender body in an infinite fluid with the kernel (23a) governing longitudinal interactions along the body. The last term in (22a) can therefore be interpreted as the free-surface correction, and the corresponding kernel (23b) can be derived in a direct manner by subtracting (15) from (14), following the approach of Newman (1964). The equivalence of (22a) and (22b) follows in a straightforward manner by partial integration with the restriction that the source strength vanishes at the body ends.

The outer and inner solutions are matched by equating (8), (12) and (22). The terms proportional to log r match automatically, since the same source strength has been anticipated for the two solutions. The remaining terms, of order one, determine the "constant" $f(x)$ in the inner solution as the sum of the last two terms in (22a). It follows that the inner solution (8) can be expressed in the alternative forms

$$\phi = \phi^{(2D)}(y,z;x) + \frac{1}{4\pi}\int_0^1 \sigma' L_o + \frac{K}{4\pi}\int_0^1 \sigma(\xi)\,L_1(Kx-K\xi)\,d\xi\,, \quad \dagger \qquad (24a)$$

or

$$\phi = \phi^{(2D)}(y,z;x) + \frac{1}{2\pi}\,\sigma(x)\left[\log(K) + \gamma\pi i\right] + \frac{1}{4\pi}\int_0^1 \sigma'(\xi)\,L_2(Kx-K\xi)\,d\xi\,. \qquad (24b)$$

The potential $\phi^{(2D)}$ is the solution of the two-dimensional Laplace equation (7), the rigid free-surface condition (11), and the body-boundary condition (2). The arbitrary constant in this solution is determined uniquely by the requirement that (12) holds at large distances from the body section, with an error $o(1)$. The source strength σ is determined by continuity, as twice the product of the local waterplane beam and vertical velocity. The only effects of the free surface are on the second term of (24a).

There are two practical objections to the above theory. One is that waves exist only in the outer region. The other is that the leading-order force opposing the motion of the ship is hydrostatic, proportional to the waterplane area and hence $O(\varepsilon)$. By comparison, the hydrodynamic force and the inertial force due to the mass of the ship are $O(\varepsilon^2)$. Thus, to leading order, there is no resonance

† See errata, Page 115.

of the pitch and heave motions. A simple dimensional argument can be used following Newman (1977), to show that the resonant frequencies are of order $(g/B)^{1/2}=O(\varepsilon^{-1/2})$.

These defects in the long-wavelength slender-body theory can be overcome by including higher-order terms in ε, following Newman and Tuck (1964). Alternatively we can revise our assumption regarding the order of magnitude of the frequency, and treat the short-wavelength problem where $\lambda/L=O(\varepsilon)$ or $K=O(\varepsilon^{-1})$.

Short Wavelengths, $K=O(\varepsilon^{-1})$

With $K=O(\varepsilon^{-1})$, both terms in the free-surface boundary condition (3) are of the same order of magnitude in the inner region. Thus we anticipate significant wave effects in the inner region adjacent to the body. The same is true in the outer region, with the assumptions that radiated waves are present, of wavenumber K, and that the gradient is proportional to K. Thus, in the short-wavelength regime, the free-surface condition (3) applies in *both* regions.

Since the boundary-value problem for the outer solution is unchanged from the long wavelength case, the solution can be constructed in an identical manner. Restricting our attention to vertical motions of ships with symmetry about the plane $y=0$, the outer solution is given by a distribution (16) of three-dimensional sources (14), and the Fourier transform of the outer solution is given by (18). Only the transform G^* must be reevaluated, with $K=O(\varepsilon^{-1})$. (Throughout we assume that $k=O(1)$, or that longitudinal variations of the quantities transformed are governed by the ship's length as opposed to the wavenumber. This assumption is not valid in the far field, where radiated waves of large wavenumber K propagate in all radial directions. However the Fourier transformed quantities are to be used only in the matching region where this assumption is more appropriate.)

For $K>>1$, the integral (20) for G^* can be approximated by deforming the contour of integration, following Ogilvie and Tuck (1969) and Faltinsen (1971). With branch cuts of the square-root function on the imaginary axis between $\pm ik$ and infinity, the contour of integration can be deformed to the upper (lower) half plane for positive (negative) values of y. The resulting integral is $O(1/K)$, and the dominant contribution is from the residue. (Recall from (6) that the imaginary part of K is a small negative quantity, and thus the poles of (20) are adjacent to the real axis in the second and fourth quadrants.) The contribution from the residue gives

$$G^* \cong -2\pi i \frac{\exp\,[Kz-i|y|(K^2-k^2)^{1/2}]}{(1-k^2/K^2)^{1/2}}$$

$$\cong -2\pi i \exp\,[Kz-iK|y|] \quad . \qquad (25)$$

Using the last form of (25), which is valid for $K>>|k|$, the inverse transform of (18) is given in the form

$$\phi = \frac{1}{2}\, i\sigma(x)\exp\,(Kz-iK|y|) \quad . \qquad (26)$$

This is the appropriate inner expansion of the outer solution for the short-wavelength case.

The inner solution is governed by the two-dimensional Laplace equation (7), the body-boundary condition (2), and the free-surface condition (3). Since the matching requirement (26) is mathematically identical to a two-dimensional radiation condition, the desired inner solution corresponds to the forced motion of the profile C, with the flow constrained to the plane x=constant. This problem has been studied extensively, for a variety of body profiles. A compendium of results is given by Vugts (1968). Various computer programs now exist for solving this problem with arbitrary body profile C, as described by Chapman (1977).

With the inner solution specified in this

manner, the problem is completely solved. Since the free-surface condition (3) does not allow an arbitrary constant to be added to the inner solution, no interaction exists between different sections of the ship. Thus we recover the strip theory of ship motions, with the flow at each section of the ship independent of the shape or motion at adjacent sections.

As noted in the Introduction, the last conclusion is cause for mixed feelings. On the positive side, it provides a rational basis for the strip theory, which originally was introduced to ship hydrodynamics in a heuristic manner. On the other hand, we began with a fully three-dimensional problem and it is disappointing to find that in the final result three-dimensional effects are absent from the inner solution. Moreover, if forward speed is included, as in Ogilvie and Tuck (1969), the leading-order solution in the inner region does not depend on the ship's forward speed.

As in the analogous situation for the long-wavelength theory, one may overcome the deficiencies of the simple strip theory by including higher-order terms in ϵ. In effect this is done in all strip theories of ship motion where forward-speed effects are included. Ogilvie and Tuck (1969) derive a consistent higher-order theory, including terms of relative order $\epsilon^{1/2}$ but neglecting $O(\epsilon)$. Included in the neglected terms are contributions to the force and moment proportional to the square of the forward speed, which are assumed generally to be of practical importance.

This situation is not entirely satisfactory. As has been stated aptly by Ogilvie (1976) in the context of wave-resistance theory, "we are more likely to make practical progress by developing a better linear theory than by devising second-order corrections to an inadequate linear theory".

A Composite Approximation

An "interpolation" between the two theories outlined above has been constructed by Maruo (1970), in a similar manner to the additive composition of matched asymptotic expansions.* First we note that the inner solutions for $K=O(1)$ and $K=O(\epsilon^{-1})$ possess a common "overlap" expansion. Thus the formal approximation of (24) for *large* K is equal to the approximation of the strip-theory potential for *small* K. The former result is obtained simply by noting that the kernel L_2, defined by (23c), vanishes for large values of its argument. Thus the large-K approximation of (24) is given by

$$\lim_{K\to\infty} \phi_{(K=O(1))} = \phi^{(2D)} + \frac{1}{2\pi}\sigma(x)\,[\log(K)+\gamma+\pi i]. \qquad (27)$$

The small-K approximation of the strip-theory potential can be obtained by expressing the solution as a source plus wave-free potentials, following Ursell (1968), and noting that the second term of (27) is the low-frequency limit of the source potential, minus the $\log(r)$ term in $\phi^{(2D)}$.

Following the rules for additive composition, we add the two inner solutions and subtract their common limit (27). It follows that $+\frac{1}{4\pi}\int \sigma' L_0$

$$\phi = \phi_{strip} + \frac{K}{4\pi}\int_0^1 \sigma(\xi)L_1(Kx-K\xi)\,d\xi \quad +$$

$$-\frac{1}{2\pi}\sigma(x)\,[\log(K)+\gamma+\pi i]$$

$$= \phi_{strip} + \frac{1}{4\pi}\int_0^1 \sigma(\xi)L_2(Kx-K\xi)\,d\xi \quad . \qquad (28)$$

This is a composite inner solution, with the property that it contains the $K=O(1)$ and $K=O(\epsilon^{-1})$ results as limiting cases. However (28) does not satisfy the inner boundary-value problem for all values of K, since the interaction terms added to

+ See errata, Page 115.

* Computations by Maruo are reported in discussion of Chang (1977). A similar approach is described by Tuck (1966).

the strip-theory potential ϕ_{strip} violate the free-surface condition (3).

In the interpolation theory of Maruo (1970), the interaction terms of (28) are multiplied by a higher-order factor (1+Kz). As a result the free-surface condition (3) is satisfied. However, the uniformity of this correction is doubtful for large K, and a troublesome term which results is ignored in Maruo's computation of the hydrodynamic pressure force.

A Unified Solution

To overcome the deficiencies of the long- and short-wavelength theories, it is logical to seek a "unified" theory which embraces both as special cases but is valid throughout the wavenumber regime $O(1) \leq K \leq O(\varepsilon^{-1})$. To ensure that all leading-order ingredients of the two limiting problems are included, the guiding priciple is that boundary-value problems are formulated in the inner and outer regions such that *for all relevant values of the wavenumber no terms of leading order in the slenderness parameter are neglected.*

Proceeding on this basis for the ship-motion problem, the outer problem is unchanged, but in the inner region both terms of the free-surface condition (3) are retained in the "inconsistent" manner suggested by Ogilvie (1974). The strip-theory potential of the short-wavelength regime is a particular solution of the resulting inner problem. However for arbitrary wavelengths it is not possible to match this solution with the outer potential. Indeed, for long wavelengths, an arbitrary additive constant must be allowed in the inner solution, as in (8). Since the free-surface condition (3) does not permit such a constant to be added to the inner solution, a nontrivial homogeneous solution is required.

The only solutions of the homogeneous problem in the inner region are the velocity potentials of the diffraction problem, for scattering of incident waves moving past the fixed body in the positive and negative directions. The appropriate solution symmetrical about $y=0$ is composed of two equal and opposite incident waves, or an "incident standing wave". This homogeneous solution violates the two-dimensional radiation condition but, from (26), that objection applies only for short wavelengths. As a result, we anticipate that the contribution to the inner potential from the homogeneous solution must vanish in the short-wavelength regime. More generally, a non-zero contribution from the homogeneous solution is both necessary and appropriate to match with the outer solution.

Thus, the unified slender-body theory for ship motions in calm water is based on retaining the complete free-surface condition (3) in both the inner and outer regions. The inner problem is simplified principally by imposing the two-dimensional Laplace equation (7), which is valid for all wavenumbers provided waves in the inner region are parallel to the x-axis. The inner solution is assumed to consist of a linear combination of the two-dimensional strip-theory potential, corresponding to forced motion of the body, and of the two-dimensional homogeneous solution for diffraction of incident standing waves by the fixed body. For the outer solution a longitudinal distribution (16) of three-dimensional sources (14) is assumed. The resulting inner and outer solutions can be matched, in a suitable overlap region $B \ll y \ll L$.

This unified theory of ship motions can be compared to the modified strip theory of Grim (1960). Grim solves the two-dimensional problem with appropriate singularities, and assumes a corresponding distribution of three-dimensional singularities with the same coefficients. To correct the resulting error in the body boundary condition, due to the change from two to three dimensions, a longitudinal diffraction solution is introduced. Here the incident waves are associated with radiation along the hull from the three-

dimensional wave source. Grim anticipates that this "longitudinal diffraction problem" can not be realized physically, as has been confirmed mathematically by Ursell (1975).

Aside from the lack of a systematic formalism based on asymptotic approximations, and the introduction of additional assumptions by Grim to simplify his computations, the principal distinction with the present unified theory is that we supplement our inner solution with a diffraction potential for *transverse* incident waves. The resulting solution is strictly two-dimensional, and well defined both mathematically and physically. The expected longitudinal interactions along the hull arise indirectly from matching with an outer solution.

The analysis of ship motions based on the unified slender-body theory is now in progress, both with and without forward speed. Rather than pursuing either study here in detail, I shall describe instead a simpler but analogous problem where the same approach can be illustrated more explicitly.

3. AN ANALOGOUS ACOUSTIC PROBLEM

Let us consider a slender axisymmetric body, of radius $r=r_o(x)$, which radiates acoustic waves in the surrounding medium. With the usual assumptions of linearized acoustics, the Laplace equation (1) is replaced by the reduced wave equation

$$(\nabla^2 + K^2)\,\phi = 0 \quad , \qquad (29)$$

where $K=\omega L/c$ is the nondimensional wavenumber and c is the speed of sound. The body boundary condition (2) is unchanged, and the radiation condition of outgoing waves at infinity can be replaced by the complex wavenumber (6).

For simplicity we assume axisymmetric forced motions of the body, with normal velocity $V(x)$. The leading-order boundary condition on the body takes the form

$$\phi_r = -V(x), \text{ on } r = r_o(x). \qquad (30)$$

The inner region is defined by the scale of the maximum body radius, $r=O(\varepsilon)$. Here a coordinate stretching argument can be used to reduce (29) to its two-dimensional form

$$\phi_{yy} + \phi_{zz} + K^2\phi = 0 \quad , \qquad (31)$$

with the error a factor $1+O(\varepsilon^2)$ as in (7). The most general axisymmetric solution of (31) is

$$\phi = a_1(x)\, H_o^{(1)}(Kr) + a_2(x)\, H_o^{(2)}(Kr) \quad , \qquad (32)$$

where a_1 and a_2 are coefficients which may depend on x, and $H_o^{(1,2)}$ denote the Hankel functions $J_o \pm iY_o$. With the assumed time dependence $e^{i\omega t}$, $H_o^{(1)}$ and $H_o^{(2)}$ represent incoming and outgoing waves, respectively, for large Kr.

The outer solution is a longitudinal distribution of acoustic sources along the body axis. Thus, as in (16)

$$\phi = -\frac{1}{4\pi}\int_0^1 \sigma(\xi)\, G(x-\xi,r)\, d\xi \quad , \qquad (33)$$

where

$$G(x,r) = \frac{e^{-iK(x^2+r^2)^{1/2}}}{(x^2+r^2)^{1/2}} \quad . \qquad (34)$$

Fourier transforms can be used as in Section 2, to derive the inner expansion of the outer solution, for purposes of matching with (32). Here we follow a modified approach, matching the transforms of both the inner and outer solutions. For this purpose the transform of the inner solution (32) is

$$\phi^* = a_1^*\, H_o^{(1)}(Kr) + a_2^*\, H_o^{(2)}(Kr) \quad . \qquad (35)$$

Using the convolution theorem (18), evaluating G^* from (34), and relating this to the integral representation

of the Hankel function, the transform of the outer solution is

$$\phi^* = -\frac{1}{4\pi}\sigma^* \begin{Bmatrix} 2\,K_o(r\sqrt{k^2-K^2}) \\ -\pi i\, H_o^{(2)}(r\sqrt{K^2-k^2}) \end{Bmatrix} \text{ for } \{|k| \gtrless K\}. \tag{36}$$

Matching now can be performed in the transformed domain, by equating (35) and (36) for a suitable range of r in the overlap domain $r_o \ll r \ll 1$. As in Section 2, we assume that $k=O(1)$, and $\epsilon \ll kr \ll 1$ in the matching region.

Short Wavelengths, $K=O(\epsilon^{-1})$

For short waves, where $K \gg |k|$, the second form of (36) applies. After expanding for small values of $|k|/K$, the transformed outer solution is

$$\phi^* \cong -\frac{1}{4\pi}\sigma^* \left[H_o^{(2)}(Kr) + \frac{1}{2}(k^2r^2/Kr)H_1^{(2)}(Kr)\right], \tag{37}$$

where the error is a factor $1+O(k^4r^2/K^2)$. Equations (35) and (37) can be matched to leading order with the results

$$a_1^*=0 \quad , \tag{38}$$

$$a_2^* = \frac{1}{4} i\sigma^* \quad , \tag{39}$$

and the error is a factor $1+O(k^2r/K)$. Transforming (38-39) determines the coefficients

$$a_1 = 0 \quad , \tag{40}$$

$$a_2 = \frac{1}{4} i\sigma \quad . \tag{41}$$

Thus, the inner solution for short wavelengths is given to leading order by

$$\phi = a_2(x)\, H_o^{(2)}(Kr) = \frac{V(x)}{K\,H_1^{(2)}(Kr_o)}\, H_o^{(2)}(Kr), \tag{42}$$

where the last equality follows from the boundary condition (30). This is the "strip theory" result, with the inner solution identical to the two-dimensional radiation from a circular cylinder. Since $a_1=0$, the two-dimensional radiation condition is satisfied in an analogous manner to the strip theory of ship motions.

Long Wavelengths, $K=O(1)$

For $(k,K)=O(1)$, and small r, the Hankel functions in (35) and (36) can be approximated from the leading terms in their infinite-series representations. Matching the two transformed solutions after this is done gives the result

$$(a_1^*+a_2^*)(1-\frac{1}{4}K^2r^2) + \frac{2}{\pi} i(a_1^*-a_2^*)[1-\frac{1}{4}K^2r^2]$$
$$\left[\log(\frac{1}{2}Kr) + \gamma + \frac{1}{4}K^2r^2\right]$$
$$\cong \frac{1}{2\pi}\sigma^* \left[1+\frac{1}{4}(k^2-K^2)r^2\right] \cdot$$
$$\left[\log(\frac{1}{2}Kr) + \gamma + \frac{1}{2}\log(|k^2/K^2-1|)\right.$$
$$\left. -\frac{1}{4}(k^2-K^2)r^2 + \begin{Bmatrix} 0 \\ \pi i/2 \end{Bmatrix}\right], \text{ for } \{|k| \gtrless K\} \quad . \tag{43}$$

Here $\gamma = 0.577...$ is Euler's constant, and the error in (43) is a factor $1+O((kr)^4,(Kr)^4)$. To leading order for small r, it follows that

$$a_1^* + a_2^* = \frac{1}{4\pi}\sigma^* \begin{Bmatrix} \log(k^2/K^2-1) \\ \log(1-k^2/K^2) + \pi i \end{Bmatrix} , \tag{44}$$

$$a_1^* - a_2^* = -\frac{1}{4} i\sigma^* \quad , \tag{45}$$

where the error is a factor $1+O((kr)^2,(Kr)^4)$.

The inverse transform of (45) is simply

$$a_1 - a_2 = -\frac{1}{4} i\sigma \quad . \tag{46}$$

The inverse transform of the factor in braces in (44) can be related to the exponential integral, after a contour integration in the complex k-plane with branch cuts on the real axis for $|k|>K$. Another application of the convolution theorem then gives the transform of (44),

$$a_1 + a_2 = \frac{1}{4} i\sigma + \frac{1}{4\pi} \int_0^1 \sigma'(\xi) L(Kx-K\xi))d\xi \; . \tag{47}$$

Here the kernel L is defined by

$$L(u) = E_1(i|u|) \; \mathrm{sgn}\; (u) \quad , \tag{48}$$

and

$$E_1(z) = \int_z^\infty \frac{e^{-t}}{t} dt \tag{49}$$

is the exponential integral .

After solving (46) and (47) for the coefficients a_1 and a_2, and substituting in (32), the inner solution can be expressed in the form

$$\phi = \frac{1}{4} i\sigma(x) \; H_o^{(2)}(Kr) + \frac{1}{4\pi} J_o(Kr) \int_0^1 \sigma'(\xi) L(Kx-K\xi))d\xi . \tag{50}$$

Finally, if the Bessel functions in (50) are expanded for small Kr, it follows that

$$\phi = \frac{1}{2\pi} \sigma(x) \left[\log(\tfrac{1}{2} Kr) + \gamma + \tfrac{\pi}{2} i\right] + \frac{1}{4\pi} \int_0^1 \sigma'(\xi) \; L(Kx-K\xi))d\xi \; . \tag{51}$$

At this stage the boundary condition (30) can be used to determine the source strength

$$\sigma(x) = -2\pi r_o(x) V(x) \quad , \tag{52}$$

and the inner solution is completed.

The outer solution and far-field radiation patterns can be derived by substituting (52) in (33). This aspect of the long-wavelength problem has been studied by Pond (1966), who deduces the source strength (52) in a global manner without consideration of the inner solution for the velocity potential. Chertock (1975) has presented a more general slender-body theory for the long-wavelength case, starting with an integral equation for the exact three-dimensional solution and using physical arguments to approximate the three-dimensional singularity distribution. No attempt is made to reduce the inner solution to the form (51).

For *very* long wavelengths, $K<<1$, the kernel (48) can be approximated from the infinite-series expansion of (49). In the limit $K \to 0$, (51) tends to the classical inner solution for an unbounded incompressible fluid,

$$\phi = \frac{1}{2\pi} \sigma(x) \log r + \frac{1}{4\pi} \int_0^1 \sigma'(\xi) \; \log(2|x-\xi|) \, \mathrm{sgn}(\xi - x) d\xi \; . \tag{53}$$

This is consistent with the corresponding limit of (22) .

A Composite Approximation

The inner solutions (42) and (51) are valid in the mutually exclusive regimes $K=O(\varepsilon^{-1})$ and $K=O(1)$, respectively. These solutions possess a common approximation in the "overlap region" $1<<K<<\varepsilon^{-1}$,

$$\phi \cong - r_o V \left[\log(\tfrac{1}{2} Kr) + \gamma + \tfrac{\pi}{2} i\right] \; , \tag{54}$$

which follows by assuming $Kr<<1$ or $K>>1$ in (42) or (51), respectively. Thus a composite solution can be derived by addition of (42) and (51), and subtraction of (54), in the form

$$\phi \cong \frac{V(x)}{K H_1^{(2)}(Kr_o)} H_o^{(2)}(Kr) + \frac{1}{4\pi} \int_0^1 \sigma'(\xi) L(Kx-K\xi)d\xi \quad , \tag{55}$$

with σ defined by (52). The analogous formula for the ship-motion problem is (28). Once again the composite solution is correct in the two limiting regimes where $K=O(1)$ or $O(\varepsilon^{-1})$, but the validity is not uniform throughout the intermediate range of wavenumbers since the last term in (55) violates the wave equation.

A Unified Solution

The deficiency in (55) can be overcome simply by observing that the results of our derivation for the long-wavelength case, up to and including (50), are consistent with the corresponding short-wavelength results. Thus, for $|k|/K<<1$, the Fourier transforms (44-45) yield (38-39). Similarly, since the integral in (47) tends to zero for $K>>1$, (46-47) yield the coefficients (40-41) and the inner solution (50) is dominated by the first term, proportional to $H_o^{(2)}$, in agreement with (42).

From this standpoint the inner solution (50) apparently is valid for all wavenumbers $K \leq O(\varepsilon^{-1})$, provided the source strength is determined in accordance with the boundary condition (30). This gives an integro-differential equation for the source strength,

$$i\sigma(x)\, H_1^{(2)}(Kr_o(x)) + \frac{1}{\pi} J_1(Kr_o(x)) \int_0^1 \sigma'(\xi) L(Kx-K\xi)\, d\xi = \frac{4}{K} V(x). \tag{56}$$

To confirm that (50) is the desired "unified" solution, we note that this velocity potential satisfies the two-dimensional wave equation (31) and the boundary condition (30) in the inner field; the corresponding outer solution (33) satisfies the three-dimensional wave equation (29) and the appropriate radiation condition. The only question which remains is to confirm that (33) and (50) can be matched for all possible wavenumbers, in some appropriate overlap region close to the body in the outer problem and far from the body in the inner problem.

The overlap region is expected to exist for a range of r such that $\varepsilon<<r<<1$, or when $r=O(\varepsilon^{\alpha})$, $0<\alpha<1$. Since the Fourier-transform parameter $k=O(1)$, $kr=o(1)$. However the value of Kr in the overlap domain is not restricted to this range. In particular, when the body is radiating short waves, the matching must be carried out in the far-field of the inner solution, where $Kr>>1$. Since the matching condition (43) neglects a factor $1+O(Kr)^4$, and the unified inner solution (50) derives from this condition, the possibility exists that (43) is not valid throughout the range of all wavenumbers $K \leq O(\varepsilon^{-1})$, in spite of the fact that it yields correct results in the limit $K=O(\varepsilon^{-1})$. This question can be resolved by noting that the error in (43) is a factor $1+O(Kr)^4$, whereas the error in the short-wavelength results (38-39) is a factor $1+O(k^2r/K)$. Requiring these errors to be of equal magnitude gives a condition for transition between the short- and long-wavelength matching relations,

$$k^2r/K = (Kr)^4. \tag{57}$$

If $r=O(\varepsilon^{\alpha})$, the transitional wavenumber is

$$K = \varepsilon^{-3\alpha/5}, \tag{58}$$

and the error in matching is a factor $1+O(\varepsilon^{8\alpha/5})$. Thus the matching error will be $O(\varepsilon)$ if the overlap region is defined by $r=O(\varepsilon^{5/8})$ at the transitional wavenumber $K = \varepsilon^{-3/8}$. A more complete analysis is necessary to prove that the full solution is accurate to $O(\varepsilon)$ with this particular choice of the overlap region. Neverthe-less it is apparent that the relative

error in the unified inner solution (30) is $o(1)$ for all wavenumbers.

The unified solution (30) can be compared with the composite approximation (35). The latter result is equivalent to replacing the Bessel function $J_0(Kr)$ by its limit of unity, in the second term of (50), and determining the source strength σ in each term of (50) from (42) and (52) respectively.

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Errata

The term in (22a) involving the kernel L_O has been deleted from (24a) and (28). To correct this error the second line of (22a) should be inserted before the plus sign in (24a), and before the first plus sign in (28).